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Capital Allocation in Financial Firms

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Abstract

This paper develops a theory of capital allocation in opaque financial intermediaries. The model endogenizes risk management and capital structure decisions, and it provides a simple setting within which to address questions relating to capital budgeting, performance measurement, and employee compensation. It provides a theoretical foundation for understanding the appropriate use, and misuse, of the widely-employed RAROC methodology.

The main implications of the model are as follows:

- Projects should be valued by calculating the net present value of cash flows using market-determined discount rates, and subtracting a deadweight cost of capital that is related to the project's marginal contribution to firm-wide risk.

- Diversification across business units reduces the firm's deadweight cost of risk capital. The diversified firm thus faces a larger investment opportunity set and can operate its units on a larger scale than comparable units operated stand-alone.

- Incentive compensation serves an important risk sharing function that results in managerial compensation being less performance-sensitive in units operated within a diversified firm than in units operated stand-alone.

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Introduction[1]

This paper addresses capital allocation decisions in contexts where risk capital, and the cost of risk capital, are first-order considerations. Within financial intermediaries in particular, risk capital is tightly monitored and managed, and even regulated. Risk bearing within such firms is expensive, and the cost of risk capital pervasively affects the management of the firm. The cost of risk capital enters into hurdle rates for capital budgeting, it determines how the firm manages risk and chooses its capital structure, and it affects performance measurement and employee compensation. The cost of risk capital also influences whether and how business units should be combined within a single firm.

The distinction between risk capital and other forms of capital arises naturally in financial intermediaries where many types of positions are financed with the issuance of low default risk liabilities such as insurance policies, savings deposits, repurchase agreements and swap agreements. Because these liabilities have little default risk, the economic risks of the positions (assets net of liabilities) are borne by other firm stakeholders. For example, when a bank issues insured deposits and engages in risky lending, the economic risk of the loans resides with the bank's equityholders and uninsured debtholders, and of course the insurer of deposits. Issuance of liabilities with low default risk is core to many of the businesses in which intermediaries are engaged and, within such firms, there thus typically is a separation between the funding and risk bearing functions of capital providers.[2] Capital allocation processes in financial intermediaries thus are concerned not only with decisions about which risks to bear but also with decisions to maintain a capital structure that permits default-free funding. Capital structure choices include maintaining a cash cushion and obtaining contingent capital in the form of contract guarantees, reinsurance, hedging instruments, and the like.

[1] I thank Bob Merton, George Chacko, Dwight Crane, Ben Esty, Ken Froot, Darius Palia, David Shimko, Rob Stambaugh, and Peter Tufano for stimulating conversations on the subject of capital allocation. I also thank participants in Harvard Business School's Global Financial System Project, participants in Harvard Business School's Financial Decisions and Control Summer Workshop, and participants in the Finance Seminar at the Yale School of Organization and Management.

[2] See Merton and Perold (1993) for a general discussion of the distinct functions of risk bearing and funding by capital providers.

For capital allocation decisions to involve more than a straightforward application of the Capital Asset Pricing Model, frictions must exist between the firm and the capital markets and/or in the internal management of the firm.[3] Frictions impose deadweight costs that must be covered by cash flows from investment decisions if these decisions are to be profitable. The hurdle rate for evaluating a project therefore should not only be related to the risk premia derived from the project's exposures to priced risk factors (as per the Capital Asset Pricing Model) but should also reflect a measure of the deadweight costs borne by the firm in undertaking the project.

In the model of this paper, I distinguish between two kinds of frictions: agency issues between the firm and the external capital markets; and agency issues between the firm and its employees. With respect to agency issues between the firm and the external capital markets, I follow Merton (1993, 1997) and Merton and Perold (1993) in viewing financial intermediaries as being special in several ways. First, as mentioned above, financial intermediaries are in credit-sensitive businesses, meaning that their customers are strongly risk-averse with respect to issuer default on contractually-promised payoffs. For example, holders of insurance policies are averse to having their loss claims be subject to the economic performance of the issuing firm, and strictly prefer to do business with a highly-rated insurer. The creditworthiness of the intermediary is crucial to its ability to write many types of contracts, and contract guarantees feature importantly in its capital structure.[4]

Second, financial firms are opaque to outsiders. These firms tend to be in businesses that depend vitally on proprietary financial technology, and that cannot be operated transparently. In addition, the balance sheets of financial firms tend to be relatively liquid, and are subject to rapid change. Financial firms thus are difficult to monitor. Guarantors face costs related to adverse selection and moral hazard, and shareholders bear "free cash

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[3] Absent imperfections, the Modigliani Miller (1958) theorem applies and the price of risk is determined by the capital market equilibrium. Intermediaries in any event do not exist in a frictionless capital market, and if they did, there would be little need for traditional risk management as firms costlessly would be able to hold large amounts of equity.

[4] This is not to say that firms with poor credit ratings cannot issue essentially default-free liabilities. For example, by posting collateral, even individuals can trade futures contracts. See Merton and Bodie (1992) for further discussion.

flow" agency costs (Jensen (1986).) It is common also for shareholders to incur deadweight costs related to double taxation (as is the case with C Corporations in the United States.)

Together, these information, agency and tax costs represent a layer of "external" deadweight costs that negatively affect the valuation of projects and businesses. In the model of this paper, the value of these deadweight costs is increasing in the total risk of the firm. All else equal, this creates a preference for decisions that reduce firm-wide risk. The preference for risk reduction will be reflected in capital budgeting rules and also in the risk management and capital structure decisions of the firm. Risk also will play a role in performance measurement and in the determination of employee compensation contracts. The model developed here represents an extension of the risk-capital approach of Merton and Perold (1993), and bears important similarities to the RAROC (Risk-Adjusted Return on Capital) class of approaches employed in many financial institutions. The model's close resemblance to RAROC makes it straightforward to see the flaws as well as strengths of some of the traditional applications of that method.

This model of agency costs between the firm and the external capital markets shares many of the same goals as Froot and Stein (1998), who apply the corporate risk-management model of Froot, Scharfstein and Stein (1993) to capital budgeting decisions in financial institutions. In their model, the firm makes an investment decision in one period, and its uncertain payoffs affect the firm's need to raise costly external funds in the next period. The model is thus driven by ex post penalties incurred by the firm in the event of a cash shortfall. Froot and Stein obtain many of the same general prescriptions as discussed qualitatively in Merton and Perold (1993), and as obtained more formally in the first part of this paper. However, Froot and Stein's model parameters are difficult to directly associate with the variables and controls—such as risk capital—that seem to be the main focus of attention in financial institutions. In contrast, the model of this paper is premised on the view that, in order to be in business in the first place, the firm must be organized to provide guarantees of its performance on customer contracts. The model seeks to explicitly account for the ex ante deadweight costs associated with funding these guarantees.

The paper next explores the implications of costly risk capital for the design of performance-based compensation contracts. Financial intermediaries have the important feature that information vital to the success of many of their core businesses tends to be private at the division level, or even at the level of individual employees. For example, when a client calls an intermediary's block trading desk to unload a large position in IBM shares, the block desk will quote the client a price conditioned on factors such as recent and impending news pertaining to IBM, the likely motivation of the seller, recent trading volume and stock price volatility, and estimated current market depth.[5] There is little the firm can do to assess the quality of any one such pricing decision by its block desk. Firms thus try to devise incentive-compatible schemes, the most important aspect of which is performance-related pay. It is common for key employees in financial firms to earn many times their base salaries in performance-related bonuses. Moreover, these bonuses often represent a substantial fraction of firm profits before payouts to employees. However, the use of performance-based compensation is not a first best solution as it shifts risk to employees and in so doing introduces additional "internal" deadweight costs in the form of employee risk aversion.

To extend the model to take into account performance-based compensation, I employ the standard principal-agent framework in which management is risk averse, managerial "effort" is not observable, and management incurs disutility of effort. In this now two-layer agency cost model, efficient compensation contracts involve a tradeoff between "external" deadweight costs arising from agency issues between the firm and the external capital markets and "internal" deadweight costs derived from the firm having to provide costly incentives to management. The key insight of the model is that performance-based compensation has the effect of management sharing in the risk of the firm, thus reducing the risk borne by external constituencies and hence reducing external deadweight costs. However, this occurs at the expense of increased costs relating to managerial risk aversion. The optimal performance sensitivity of compensation thus depends on the relative importance of externally- versus internally-derived deadweight costs.

[5] For a case study on block trading, see Perold (1984).

The model of the paper yields a number of implications for the efficiency of combining business units within a diversified firm versus operating the units stand-alone. First, because the value of the firm is decreasing in the total risk of the firm, diversification across imperfectly-correlated business units reduces a firm's external deadweight cost of risk capital. The diversified firm therefore has lower required returns for investment decisions. With decreasing returns to scale, a unit housed within a diversified firm can undertake projects that would be unprofitable if that unit were operated stand-alone. Units within a diversified firm thus face a larger investment opportunity sets and can operate on a larger scale than stand-alone.

The model also implies that it is inefficient to operate a transparent business within an opaque financial firm. (This point is also made in Merton (1993).) A transparent business—for example, an S&P 500 index fund—bears no deadweight costs of risk capital. Operating a transparent business within an opaque intermediary does result in lower total firm risk than operating that unit outside of the firm. However, its inclusion changes a transparent business into an opaque business, which leads to an increase in the deadweight cost of firm-wide risk capital. Only if the correlation were negative would including a transparent business at the margin reduce total firm risk, and thus reduce the cost of firm-wide risk capital. In that case, that business performs the same function as a hedging instrument. The main point here is that, in this model, diversification is beneficial across intermediation businesses that face similar agency costs of risk capital.

With decreasing returns to scale, the model of this paper also illustrates a noteworthy accounting effect relevant to the empirical literature on corporate diversification. Studies find that diversified firms are valued at discounts relative to non-diversified firms in their industries.[6] As shown in the body of the paper, with decreasing returns to scale, the diversified firm's net present value per unit of scale can be less than that of the units operated stand-alone even though the diversified firm is more valuable than the sum of the units

[6] See Campa and Kedia (1999) and Palia (1999) for recent summaries of this literature.

operated standalone. In other words, the empirically measured discounts may be spurious and be an artifact of not adjusting for the expanded investment opportunity sets faced by diversified firms.[7]

Another diversification-related implication of the model is that the performance sensitivity of the compensation of division-level management should be lower in diversified firms than in divisions operated stand-alone. The model allows for division-level management to be compensated as a function of division-level profits as well as firm-wide profits. The effect (described earlier) of performance-based compensation leading to risk sharing with employees thus occurs at both levels. Because diversification across divisions reduces total firm risk capital, this benefit of risk sharing with employees is less pronounced for diversified firms. Therefore, the sensitivity of divisional manager compensation to both divisional and firm-wide performance should be lower in diversified firms than in business units operated stand-alone.

Palia (1999) reports evidence consistent with the pay-performance sensitivity predictions of this model. In a broad sample of diversified and focused firms, he finds that division managers of diversified firms receive significantly lower annual option grants and own a much smaller fraction of the firm than the management of single-segment firms. Palia obtains the same findings for CEOs, as do Anderson, Bates, Cizjak and Lemmon (1988). An important caveat of course is that the samples in these papers are not restricted to financial firms. Whether and how the pay-performance effects modeled in this paper apply to non-financial firms is the subject of further research.

In concluding this introduction, it should be pointed out that the notion that diversification may have value to firms generally is of course not a new idea. Theoretical models of the benefits of internal capital markets for firms include Williamson (1975), Gertner, Scharfstein and Stein (1994), and Stein (1997), although the focus of these papers is not to establish a corporate price of risk per se. Theories where risk matters structurally

[7] For example, in a sample of non-financial firms, Campa and Kedia (1999) find that the diversification discount drops, and sometimes turns into a premium, after controlling for the endogeneity of firms' diversification decisions.

include Lewellen (1971); Stulz (1990); Froot, Scharfstein and Stein (1993); and Froot and Stein (1998) (discussed earlier.) Empirically, Berger, Cummins, Weiss and Zi (2000) find that there are diversification-related benefits in the insurance industry although their hypothesis concerns scope economies rather than risk reduction. Hadlock, Ryngaert and Thomas (1998) find that diversified firms have lower costs of issuing public equity and interpret the finding as evidence that diversification reduces the cost of asymmetric information.

There is also a vast empirical and theoretical literature that takes the opposite point of view—that corporate diversification creates its own deadweight costs. Inefficiencies associated with corporate diversification include diminishing returns to scope—where the firm's monitoring effectiveness declines with the number of unrelated projects or businesses (e.g., Stein (1997)); private benefits that accrue to senior management and that are derived from firm size; internal power struggles (Rajan, Servaes and Zingales (1997)); and influence peddling (Wulf (2000).) Diversified firms also might be less efficient than stand-alone firms in motivating employees to innovate and accumulate human capital (e.g., Hart and Moore (1990) and Rotemberg and Saloner (1994)). The model of this paper abstracts away from these frictions. Whether opaque financial intermediaries experience a reduction in their cost of risk capital sufficient to outweigh the inefficiencies associated with corporate diversification is a question outside the scope of this paper.

The next two sections of the paper provide institutional background on risk capital to motivate the model. These observations are based partly on field research conducted at major banks and investment banks as part of the Global Financial System Project at Harvard Business School. The model is then formulated, and its normative implications discussed.

Risk Capital

The risk capital of a position, or of a business, measures its potential for loss.[8] A standard practice is to define risk capital in terms of the tail of the loss distribution, so-called value-at-risk. Value-at-risk which measures the worst-case loss the firm expects to incur in a high percentage (for example, 99%) of outcomes over a given period. Merton and Perold (1993) define risk capital in economic terms as *the smallest amount that can be invested to insure the value of the firm's net assets*[9] *against a loss in value relative to a risk-free investment.*[10] According to this definition, the risk capital of a long U.S. Treasury bond position, for example, is the value of a put option with strike price equal to the forward price of the bonds.[11] In general, the put option accounts for the full distribution of losses whereas value-at-risk ignores the magnitude of outcomes conditional on being in the extreme tail of the distribution. When returns are normally distributed, the value of such a put option is approximately proportional to the standard deviation of the return on the bond, and thus is approximately proportional to value-at-risk.[12]

As already pointed out, firms in multiple businesses generally require less risk capital than would the collection of those businesses operated on a stand-alone basis.[13] The diversification benefit can be large, depending on the correlations among the profits of the constituent businesses. The magnitude of the effect is illustrated below using data obtained from a major New York investment bank. The investment bank made available summary statistics of the 1996 daily revenues (net of interest expense but not net of compensation or

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[8] In practice, financial firms tend to focus not only on risk capital, but also "working capital," which is the amount of funding required to establish and maintain balance sheet positions; and "regulatory capital," which is an accounting measure of risk capital as defined by regulatory authorities. Working capital and regulatory capital are highly institution and instrument specific, and the model of this paper abstracts away from the importance of these types of capital.

[9] Net assets refers to gross assets minus customer liabilities (swaps, insurance contracts, etc.), valued as if these liabilities are default-free.

[10] Ait-Sahalia and Lo (1998) suggest measuring "economic risk capital" in terms of value-at-risk calculated with respect to the risk-neutral probability distribution.

[11] The forward price is the current price plus interest calculated at the risk-free rate minus any coupons paid in the interim.

[12] When the strike price is equal to the forward price of a security, the Black-Scholes (1973) formula evaluates to $2N(\frac{1}{2}\sigma\sqrt{t}) - 1$, which is approximately equal to $0.4\sigma\sqrt{t}$, for small $\sigma\sqrt{t}$. N(.) is the cumulative standard normal distribution function.

[13] A side point: There are similar "diversification" effects for regulatory capital. For example, one business may require $100 of regulatory capital on odd days, while another may require $100 of regulatory capital on even days. In combination, these two businesses can be funded with only $100 of investor capital, as opposed to $200 stand-alone.

other non-compensation expenses) of its more than 20 trading businesses. The business units were of varying size, and together accounted for a significant fraction the firm's total profits. **Table 1** shows the correlations between the daily trading revenues of the units, summed within product segments: interest-rate products, equity products, foreign-exchange products, and commodity products. Within each segment, some units traded physical instruments, while others issued and traded derivative instruments. **Table 2** shows the distribution of correlations between the daily trading revenues of the units within each segment. Perhaps surprisingly, the correlations within and between product segments are very low, and some correlations are even negative.

Table 1: Correlations Between Trading Revenues of Businesses Within Major Product Segments

	Interest-Rate Products	Equity Products	Foreign-Exchange Products	Commodity Products
Interest-Rate Products	1.000			
Equity Products	0.135	1.000		
Foreign-Exchange Products	0.053	-0.111	1.000	
Commodity Products	0.057	-0.007	-0.002	1.000

Table 2: Distribution of Correlations Between Unit Trading Revenues

	All Trading Businesses	Interest-Rate Products	Equity Products	Foreign-Exchange Products	Commodity Products
Range of correlations:					
Low	-0.430	-0.078	-0.430	-0.014	-0.124
High	0.401	0.191	0.284	0.401	0.075
Average correlation	0.065	0.126	0.125	0.202	0.009

Table 3 shows the diversification effect by calculating the standard deviation of trading revenues of a combination of units divided by the sum of the standard deviations of individual unit trading revenues. As might be expected from the low correlations, the effect

can be dramatic: Segment risk capital ranges from 45.2% to 81.0%[14] of the stand-alone risk capital; firm-wide risk capital is 60.0% of stand-alone risk capital at the segment level, and only 29.8% of stand-alone risk capital at the level of the individual business units.

Table 3: Diversification Effect

	All Trading Businesses	Interest-Rate Products	Equity Products	Foreign-Exchange Products	Commodity Products
Risk capital of combination vs risk capital of stand-alone *units*	29.8%	46.5%	45.2%	81.0%	58.2%
Risk capital of combination vs risk capital of stand-alone *segments*	60.0%				

Allocation of the Cost of Risk Capital

As already mentioned, if deadweight capital costs are related to risk capital—as is the case in the model of this paper—then, all else equal, opaque diversified firms should experience lower overall capital costs, and so face more profitable investment opportunities than undiversified firms. Merton (1993) and Merton and Perold (1993) point out that this positive externality should affect the choice of businesses in which the firm is engaged, and also the firm's policy towards risk management. In particular, opaque financial firms should hedge risk when this can be accomplished through the use of low-cost instruments.

The externality created by deadweight costs being a function of firm-wide risk complicates the process of capital allocation. With risk capital not being additive across the

[14] The relatively small risk reduction from diversification within foreign-exchange products is the result, principally, of the

businesses that comprise the firm, rules that fully allocate firm-wide risk capital across the constituent businesses are likely to be suboptimal. The need to manage this externality creates a role for coordinated decision-making.

The following example illustrates these points.[15] A firm is engaged in business A, and is trying to decide whether to enter business B. Each business has $100 of stand-alone risk capital. Businesses A and B have expected profits of $30 and $10, respectively, and the correlation between their profits is **zero**. Assuming that risk capital is proportional to standard deviation, the risk capital of businesses A and B combined is $141.4 (= $100\sqrt{2}$), with expected profits of $40. A full (pro-rata) allocation of risk capital would attribute $70.7 to each business. On the other hand, at the margin, each business is adding only $41.4 to required risk capital. **Table 4** below illustrates the sensitivity of the return on capital to the choice of denominator. For example, the return on capital for business B is 10%, 14% or 28%, depending on whether stand-alone risk capital, fully-allocated risk capital, or marginal risk capital, respectively, is used.[16]

lower number of units within this segment.

[15] This is a variant of an example worked out in Merton (1993).

[16] The qualitative relationships among standalone risk capital, firm-wide risk capital, and marginal risk capital, as exhibited in this example, hold generally. Merton and Perold (1993) show that the sum across business units of stand-alone risk capital is always greater than firm-wide risk capital, which in turn is always greater than the sum across business units of marginal risk capital.

Table 4: Return on Allocated Capital

	(1)	(2)	(1)÷(2)	(3)	(1)÷(3)	(4)	(1)÷(4)
Business	Expected Profits	Stand-Alone Risk Capital	Return on Capital	Fully-Allocated Risk Capital	Return on Capital	Marginal Risk Capital	Return on Capital
A	$ 30	$ 100.0	30%	$ 70.7	43%	$ 41.4	72%
B	$ 10	$ 100.0	10%	$ 70.7	14%	$ 41.4	24%
A+B	$ 40	$ 141.4	28%	$ 141.4	28%	$ 82.8	48%

How should capital be allocated so that the correct decision can be made with respect to entering business B? The answer depends on how entering business B affects the firm's deadweight cost of capital. In what follows, the firm's deadweight cost of capital is assumed to be proportional to its risk capital—specifically, 20% of risk capital. **Table 5** performs an NPV analysis by subtracting this deadweight cost of risk capital from expected profits.[17] Stand-alone, after deadweight costs, business A is profitable, but business B loses money. However, the combination of A and B makes $2 more in profits net of deadweight capital costs than just A, so that adding B to A is a good decision. To evaluate this decision, it suffices to calculate the profitability of B after assessing a 20% charge on its *marginal* risk capital of $41.4.

Table 5: Expected Profits Net of Deadweight Capital Charges

	A	**B**	**A+B**	**Incremental**
Expected Profits	$ 30	$ 10	$ 40	$ 10
Cost of risk capital (20%)	$ (20)	$ (20)	$ (28)	$ (8)
Profits after deadweight cost	$ 10	$ (10)	$ 12	$ 2

[17] For convenience of exposition, I am assuming here that the risk-free rate is zero, and that operating profits are bear no systematic risk.

Suppose next that the expected profits of business A were only $10 instead of $30. As shown below in **Table 6**, investing in business B would still be a good decision relative to remaining only in business A. However, an even better decision would be to be in neither business. This latter conclusion must be reached by considering firm-wide capital costs, and is unlikely to be arrived at through a decentralized capital allocation mechanism.

Table 6: Expected Profits Net of Deadweight Capital Charges

	A	B	A+B	Incremental
Expected profits	$ 10	$ 10	$ 20	$ 10
Cost of risk capital (20%)	$ (20)	$ (20)	$ (28)	$ (8)
Profits after deadweight cost	$ (10)	$ (10)	$ (8)	$ 2

In practice, firms in multiple businesses appear to have adopted at best ad hoc procedures for dealing with the diversification benefit of less required risk capital and commensurately lower capital costs. For example, many firms recognize the diversification benefit within business units (which are themselves viewed as diversified portfolios of individual risks) but not between business units. In effect, these firms are assuming that inter business-unit profits are perfectly correlated. There is thus not the same recognition that diversification between business units has important benefits. Firms that explicitly account for the diversification effect—whether within or between business units—differ in how they apply it to the management of the firm. Some firms base their decisions on a full allocation of firm-wide risk capital, while others base their decisions on the allocation of marginal risk capital.[18]

Firms also differ in how they determine the cost of capital. Some estimate a required return for each business as if it were stand-alone; others estimate a cost of capital for the firm overall, and then fully or partially allocate this cost to individual businesses. Approaches to estimating the cost of capital vary, including various ad hoc methods as well as standard applications of the Capital Asset Pricing Model.[19]

These divergent approaches to capital allocation appear to result at least partly from confusion about deadweight capital costs. In the absence of deadweight costs, each investment decision should be evaluated stand-alone, and the hurdle rate applicable to any project need bear no relation to the firm's "cost of capital." To illustrate, consider a transparent firm that is evaluating stand-alone projects, each of which has no systematic risk. Suppose further that the supply of good projects is correlated with the market. In this case, standard theory correctly says to value each project using the risk-free rate. Yet, the firm's cash flows will be correlated with the market, and hence the firm's stock price will contain systematic risk. Thus, the discount rate for valuing projects within the firm can be quite different from the discount rate for valuing the firm itself, and it would be a mistake to use the firm's cost of capital to evaluate individual projects. On the other hand, in opaque firms, the profits from investment decisions must cover the deadweight costs of continuing to stay in business. These are firm-wide costs that must be accounted for in the capital allocation process.

To develop normative prescriptions related to these questions, a model of a highly-stylized financial intermediary follows.

A Model of the Financial Firm

The firm is modeled here as being in business to issue custom financial contracts. To render the contracts default free, the firm holds a combination of cash and an externally-provided guarantee of its performance under these contracts. In the spirit of Merton (1989), the firm exists because it has the lowest operating costs—here assumed zero—of creating

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[18] See, for example, De Mello and Wahrenburg (1992), Wee and Lee (1995), and Litterman (1997).

[19] Examples of prescriptions for the use of the CAPM in this setting can be found in De Mello and Wahrenburg (1992), and Zaik, et al (1996).

these contracts. However, the firm bears various deadweight costs. First, the guarantor monitors the firm to protect itself against adverse selection and moral hazard. The guarantor assesses a monitoring charge assumed to be proportional to the fair market value of the guarantees.[20] Second, there are tax and free-cash flow agency costs associated with excess cash available for distribution to shareholders. These costs are assumed to be proportional to shareholder payouts.

In determining how much initial cash to hold, the firm weighs the two forms of deadweight cost: The higher the level of cash, the less coverage the firm requires in the way of contract guarantees and the lower the monitoring charges assessed by guarantors; however, the more cash the firm holds, the higher the tax and free-cash-flow agency costs associated with funds available for payouts to shareholders. The firm chooses the beginning level of cash to optimize this tradeoff. At the optimal capital structure, the firm's net present value (or, equivalently, its premium over book value) evaluates to the market value of the firm's operating profits less a deadweight cost of risk capital that is proportional to the risk capital of the firm. This convenient functional form for the value of the firm makes it straightforward to derive prescriptions for capital budgeting rules and risk management policies. It also provides a simple setting within which to model the pay for performance sensitivity of compensation contracts.

Formally, the firm is in business between t = 0 and t = 1. It engages in intermediation activities which involve issuing default-free customer liabilities at t = 0, and paying off on these liabilities at t = 1. Between t = 0 and t = 1, the firm takes offsetting hedging positions in traded securities.

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[20] The assumption of proportional agency costs serves as a convenient "reduced form." For a structural model of optimal contracting between the guarantor and the firm, see Merton (1997). Merton develops a dynamic costly value verification model in which the guarantor creates efficient incentives for management to liquidate the firm when its net assets reach zero. Operating profits are assumed normally distributed, yet the endogenously determined value of the firm is lognormally distributed. Agency costs related to the guarantee are an increasing but complex function of the total risk of the intermediary.

The firm profits by issuing its liabilities at a spread μ over fair market value. The liabilities are customized to client needs, and cannot be perfectly hedged with available traded instruments. The cumulative hedging error ("basis risk") represents the total risk of the firm.

Let L(t) be the default-free value of the firm's liabilities, issued to clients at a price of $L(0) + \mu$. An amount L(0) is invested in the hedging portfolio whose market value at t is H(t). The balance—the spread μ—is invested at the risk-free rate. The cumulative hedging error is

$$E(t) = H(t) - L(t)$$

with E(0) = 0. The firm's end-of-period operating profits, X, are given by:

$$X = \mu(1+r) + E(1)$$

where r is the one-period risk-free rate.

For convenience, it is assumed that the hedging error follows a stationary arithmetic Brownian motion process:

$$dE(t) = \alpha dt + \sigma dz$$

where the drift term, αdt, represents the risk premium for exposure of the hedging error to systematic risk ($\alpha = 0$ if there is no exposure to systematic risk). The firm's operating profits thus are normally distributed with mean $\mu(1+r) + \alpha$ and standard deviation σ. Many of the insights of this model can be obtained for more general processes.

Capital Structure

To enable it to issue default-free liabilities, the firm purchases insurance from an external credit-worthy guarantor.[21] The insurance pays any shortfall faced by the firm in settling its obligations at $t = 1$. In addition, the firm holds a cash cushion, C, which is invested at the risk-free rate. The cash cushion consists of the spread, μ, plus additional externally-raised funds, $C - \mu$. The total external investment required to create the firm is thus $C - \mu$ plus the cost of the insurance.

The firm's net assets at $t = 1$, S, is given by:

$$S = C(1+r) + E(1)$$

The shortfall to be covered by the insurance is thus:

$$S^{-} = \max\{-S,0\}.$$

In summary, the firm's balance sheet is made up of assets which consist of the hedge portfolio, the insurance, and the cash cushion; and obligations which consist of the customer liabilities and the residual shareholder claim.

Deadweight costs

The firm is opaque to outside claimants including customers and external shareholders. Deadweight costs arise in two ways. First, to avoid the costs of adverse selection and moral hazard, the insurer monitors the firm. It charges a monitoring fee in the form of an upfront proportional spread m assessed on the insurance premium. The full cost

[21] Merton and Perold (1993) discuss how this is a general specification. The risk of loss is always shared among firm claimants, including shareholders, debtholders, third-party guarantors, and customers. These claimants collectively are providers of "insurance" to the firm.

of insurance is thus $(1+m)V\{S^-\}$, where V{ } denotes the present-value operator, described below.

Second, shareholders bear direct deadweight costs such as double taxation and Jensen (1986)-type "free-cash flow" agency costs. These costs are modeled as taking the form of leakage which occurs between settlement of the firm's customer liabilities and distribution of any surplus funds to shareholders. The leakage is a fraction d of the shareholder surplus, S^+, where:

$$S^+ = \max\{S,0\}$$

Shareholder equity is therefore worth $(1-d)V\{S^+\}$.

Valuation

Since the hedge portfolio and the liabilities are initially equal in market value, the present value of the hedging error is zero. Accordingly, the present value of the firm's profits is simply equal to the spread, μ.

At t = 0, the net present value of creating the firm is given by the value of shareholders' equity less the investment required to create the firm:

$$NPV = (1-d)V\{S^+\} - (C-\mu) - (1+m)V\{S^-\}$$

which simplifies to

$$NPV = \mu - (dV\{S^+\} + mV\{S^-\})$$

The net present value thus is equal to the value of the firm's operating profits less the value of the deadweight costs.

$V\{S^+\}$ and $V\{S^-\}$ can be evaluated by taking expectations with respect to the risk neutral distribution of the hedging error, and by discounting at the risk-free rate. The risk-neutral distribution of the hedging error has standard deviation σ, and mean zero.[22] The valuations are:

$$V\{S^+\} = \sigma(n(z) + zN(z))/(1+r)$$

$$V\{S^-\} = \sigma(n(z) - zN(-z))/(1+r)$$

where $z = C(1+r)/\sigma$; n(.) and N(.) are the standard normal and cumulative standard normal distributions, respectively.

Minimization of Deadweight Costs

In this model, the higher the initial cash cushion, the lower is the monitoring cost, and the higher are the deadweight costs associated with funds available for distribution to shareholders. There is, thus, a tradeoff between the two forms of deadweight cost. Total deadweight costs are minimized when the initial cash cushion is chosen so that:

$$\text{Probability of experiencing a shortfall} = d/(d+m)$$[23]

where probability is measured with respect to the risk neutral distribution. Under the assumption of normality, the initial cash cushion, C*, that minimizes deadweight costs satisfies:

[22] I am grateful to George Chacko for discussions on the subtleties of valuing one-period normally-distributed cash flows. In particular, cash flows that are normally distributed at t = 1 with variance δ^2 can be generated, among other things, by any of an infinite number of Ornstein-Uhlenbeck processes of the form $dE = (aE+\alpha)dt + \sigma dz$ where $\delta^2 = \sigma^2(\exp(2a)-1)/2a$. $\delta^2 = \sigma^2$ when a = 0. The assumption of a = 0 restricts the equivalent martingale measure to have a one-period variance equal to the instantaneous variance.

[23] This result does not depend on the assumption of normality: Let F(.) denote the risk-neutral cumulative distribution function of the hedging error, E. Let q = probability of shortfall = $F(-C(1+r))$. Then $\partial E\{S^+\}/\partial C = (1-q)(1+r)$, and $\partial E\{S^-\}/\partial C = -q(1+r)$. At the optimum, $d\partial E\{S^+\}/\partial C + m\partial E\{S^-\}/\partial C = 0$, so that $q = d/(d+m)$.

$$C^* = Z(m/(d+m))\,\sigma/(1+r)$$

where Z(.) is the inverse of the cumulative standard normal distribution function.

This result says that when the deadweight cost of equity financing is low relative to the costs associated with adverse selection and moral hazard, the firm will partially insure through means of a cash cushion. When the profits are riskless, the firm begins with a zero cash position. That is, the firm distributes the spread, μ, to shareholders at t = 0. This way, it bears neither form of deadweight cost.

At the optimal initial cash cushion, C*, the value of deadweight costs evaluates to:

$$dV\{S^+\} + mV\{S^-\} = \frac{\sigma}{\sqrt{2\pi}}(d+m)\exp -\tfrac{1}{2}Z^2(m/(d+m))/(1+r)$$

This yields the model's main result:

Main Result

At the optimal capital structure, the net present value of creating the firm is:

$$NPV = \mu - kR$$

where:

$$R = \frac{\sigma}{\sqrt{2\pi}(1+r)}$$

$$k = (d+m)\exp -\tfrac{1}{2}Z^2(m/(d+m)).$$

R is the value of a put option on the hedging error, and is the Merton and Perold (1993) measure of risk capital.[24] The factor k is the endogenously-minimized deadweight cost of risk capital, given as a weighted sum of d and m.

This result can be restated in terms of return on risk capital. For shareholders to invest in this firm, the expected return on risk capital must satisfy $\mu/R > k$.

Numerical Example

Assume that the risk-free rate is r = 10%. The value of operating profits is μ = $150 million, and the standard deviation of profits is σ = $250 million. Based on a normal distribution, the firm's risk capital amounts to R = $91 million, using $R = \frac{\sigma}{\sqrt{2\pi}(1+r)}$. Its return on risk capital is 165% (μ/R = $150/$91).

Suppose further that there are free cash flow agency costs of d = 10%, and monitoring costs of m = 100%. The firm optimally determines its cash cushion C* = $303 million to give a probability of shortfall of 9.1% (= d/(m+d)). The tradeoff between the two types of deadweight costs is illustrated in **Figure 1**. At the optimum, the deadweight cost of risk capital is $41 million dollars on risk capital of $91 million, or k = 45.1%. The firm's return on risk capital (165%) well exceeds it required return (k = 45.1%.)

At the optimum level of initial cash, the value of the guarantee, $V\{S^-\}$, evaluates to $9.5 million. The price paid for the guarantee is $(1+m)V\{S^-\}$, which therefore equals $19 million. The investor capital required to fund the business is the initial externally-provided cash, C - μ, plus the cost of the guarantee, summing to $172 million. This is its "book value." The net present value of this investment is given by the value of expected profits less

[24] For a normally distributed variate X with mean zero, $E\{\max(-X,0)\} = \frac{\sigma}{\sqrt{2\pi}}$.

the deadweight cost of risk capital, or \$150 - \$41 = \$109 million. The firm is therefore worth a premium over book value of \$109 million, or a total of \$281 million.

Figure 2 graphs k and C* as functions of d, holding m fixed at m = 100%.[25] When the deadweight costs d and m are equal, k is simply the sum of the two costs (d+m), and the firm chooses not to hold any initial cash (C* = 0), distributing the spread, μ, to shareholders at t = 0. When there are no free cash flow agency costs (d = 0), the firm insures its customers by holding an infinitely large cash cushion ($C^* = \infty$). This way, it can avoid all deadweight costs (k = 0). And when there are no costs of moral hazard/adverse selection (m = 0), the firm also can avoid deadweight costs by buying an infinite amount of insurance coverage ($C^* = -\infty$).[26]

Application of the Model to Capital Allocation Within the Firm

The net present value of the firm, μ - kR, is the criterion that the capital allocation process should seek to maximize. This criterion becomes especially simple to apply if the deadweight costs d and m are unaffected by capital allocation decisions, for then k is a constant—i.e., total deadweight costs are proportional to risk capital. The assumption that d and m are constant will be made throughout the remainder of this paper.

Capital budgeting

With k constant, any *single* project can be evaluated by calculating the *marginal effect*, $\Delta\mu$, of the project on the value of expected operating profits, and the *marginal effect*, ΔR, of the decision on firm-wide risk capital. The decision will enhance shareholder value if the incremental return on incremental risk capital exceeds the required return on risk capital, i.e.,

[25] Note that k is a function only of d and m, and that k(d,m) = k(m,d).

[26] Interpret this as the firm issuing an infinite amount of riskless senior debt, and using the proceeds to purchase the insurance coverage needed to make this debt riskless.

$$\Delta\mu - k\Delta R > 0 \quad \text{or} \quad \Delta\mu/\Delta R > k \text{ (if } \Delta R > 0)$$

A marginal analysis would also work for multiple simultaneous decisions, provided that the decisions collectively do not result in large changes.[27] As previously discussed, marginal analyses generally will involve the allocation of less than the firm's total risk capital.[28]

This prescription can be thought of as a two-step process:

Step 1: Calculate the value of incremental profits ($\Delta\mu$). $\Delta\mu$ is equal to the expected profits of the project discounted at a market-required discount rate, such as that given by the Capital Asset Pricing Model. $\Delta\mu$ represents the value of profits associated with the project absent any deadweight costs. This step is a textbook capital budgeting analysis.

Step 2: Subtract the incremental deadweight cost of risk capital ($k\Delta R$), which here is proportional to incremental risk capital. For projects that are small relative to the firm as a whole, the incremental risk capital is approximately proportional to risk capital according to:

$$\Delta R = \beta R$$

where β is the regression coefficient of the project's profits on firm-wide profits.

The expression βR for the incremental risk capital of small project shows that projects whose profits are uncorrelated with firm-wide profits do not contribute to deadweight costs at the margin, and can be valued simply using market-required discount rates. For projects whose profits covary negatively with firm-wide profits, the marginal cost of risk capital is *negative*. Such projects represent a form of hedging, and free-up risk capital when taken on

[27] Iterating in a sequence of simultaneous but small decisions, each evaluated in terms of their marginal impact on profits and on firm-wide risk capital, is equivalent to performing the gradient algorithm for maximizing a non-linear function of N variables.

within the firm. They might be attractive even if the present value of incremental profits is negative ($\Delta\mu < 0$).

Risk management

The model's prescription for risk management is completely straightforward. Any risk that can be costlessly hedged should be hedged.[29] By definition, a costless hedge does not change the value of operating profits. Nevertheless, it reduces required risk capital and therefore lowers the firm's deadweight cost of risk capital. Costly hedges should be evaluated in the same way as any other incremental project, as described above.

Comparison with RAROC

"Standard" RAROC evaluates a project according to a required return on risk capital. The method allocates (actually, associates) an amount of risk capital proportional to a project's value-at-risk, say the 1% tail of the distribution of profits.[30] For normally-distributed profits, risk capital measured this way is proportional to the standard deviation of profits, as in the model developed in this paper. RAROC then calculates the ratio of expected future profits to allocated risk capital, and compares this ratio to a hurdle rate. As discussed earlier in the paper, there is little commonality in the way firms determine the hurdle rate.

The model of this paper is obviously similar to RAROC in so far as it relates a ratio of profits to risk capital ($\Delta\mu/\Delta R$) to a hurdle rate k. But there are crucial differences. Here, the numerator ($\Delta\mu$) is the economic value of profits—the value of profits calculated using market-based required returns. The denominator (ΔR) is the project's marginal, rather than its stand-alone, risk capital, and it moreover represents economic risk capital—the price of insuring against losses. Finally, the "hurdle rate" k measures the firm's deadweight cost of

[28] See footnote 17.
[29] Costs here refer to financial frictions such as bid-ask spreads.
[30] See Wee and Lee (1995) and Zaik et al. (1996).

risk capital. If there are no deadweight costs, this rule reduces to the standard net-present-value criterion: $\Delta\mu > 0$.

Diversified versus stand-alone firms

To evaluate the potential gains from operating multiple businesses within a firm versus on a stand-alone basis, consider the following stylization of the firm's investment opportunity set. Division i is operated on a scale S_i, where scale might be measured in terms of trading volume, the notional value of overnight derivatives positions, or the dollar value of "operating" assets held on the balance sheet. Operating profits X_i are normally distributed with present value $\mu_i(S_i)$ and standard deviation $\sigma_i S_i$, where σ_i is a constant. The standard deviation of operating profits is linear in scale, whereas expected profits are non-linear in scale. Each division exhibits decreasing returns to scale—i.e., $\mu_i(S_i)$ is concave in S_i. The correlation between the operating profits of division i and division j is ρ_{ij}, independent of scale. Firm-wide profits are additive across divisions, so that the value of firm-wide profits is $\mu_F = \Sigma\mu_i(S_i)$, and the variance of firm-wide profits is $\sigma_F^2 = \Sigma\Sigma\rho_{ij}\sigma_i\sigma_j S_i S_j$.

The capital-allocation problem here is to choose the operating scale for each business division so as to maximize net present value. The optimal allocation is determined at the point where marginal profitability equals the marginal cost of risk capital, i.e., $\mu_i'(S_i) = k\ \partial R/\partial S_i$.

To simplify further, consider the special case of symmetric divisions facing identical investment opportunity sets, and whose profits are uncorrelated ($\mu_i(.) = \mu(.)$, $\sigma_i = \sigma$, $\rho_{ij} = 0$ for all i and $j \neq i$). Since the divisions are symmetric, each will have the same optimal scale, S(N), when operated in an N-unit firm. Firm-wide risk capital then can be expressed as:

$$R = \sqrt{N}\ R_U S(N)$$

where R_U is the stand-alone risk capital of an individual business unit with scale S = 1.

Setting marginal profitability equal to the marginal cost of risk capital yields

$$\mu'(S(N)) = k\, R_U/\sqrt{N}$$

which means that diversified firms (symmetric divisions with uncorrelated profits) will take on projects whose profitability clears a hurdle that decreases in proportion to $\sqrt{N}$. Since there are decreasing returns to scale, S(N) is increasing in N, so that the individual business units will be operated on a larger scale within the diversified firm than stand-alone. In addition, the optimal scale is decreasing in risk (R_U) and in the deadweight cost (k).

The net present value per business unit can be written as

$$\text{NPV per business unit} = \mu(S(N)) - S(N)\, \mu'(S(N))$$

which is increasing in N as $\mu(.)$ is concave. Per business unit, diversified firms thus have higher dollar premiums over book value than a collection of N units operated stand-alone.

However, diversified firms likely will have lower premiums over book value <u>per unit of scale</u>. That is:

$$\text{NPV per business unit}/S(N) = \mu(S(N))/S(N) - \mu'(S(N))$$

This quantity is <u>decreasing</u> in N for N large enough if, for example, $\mu(.)$ is bounded above. In this case, diversified firms will spuriously appear to be valued at a “conglomerate discount”.

To illustrate the illusion of a conglomerate discount, consider a firm composed of N symmetric divisions with uncorrelated profits, and with

$$\mu(S) = \sqrt{S}$$

Solving for the optimal scale per business unit gives

$$S(N) = N/(2k\,R_U)^2$$

Thus, as diversification increases, the business unit scale grows proportionally in N.

The net present value per business unit evaluates to $\sqrt{N}/(4kR_U)$, versus $1/(4kR_U)$ when a unit is operated stand-alone. The per-unit net-present-value gain from within-firm diversification thus grows in proportion to $\sqrt{N}$.[31]

Finally, the net present value per unit of scale is given by $kR_U/\sqrt{N}$, which creates the appearance of a conglomerate discount. That is, the net present value multiple of scale of the diversified firm is $1/\sqrt{N}$ times that of the stand-alone firm. Of course, in this model, it is suboptimal to operate units on a stand-alone basis, and the appearance of a discount derives from the non-scalability of the investment opportunity set.

Incentive Compensation

The framework developed in this paper can be extended to questions of incentive compensation. In theory, incentive compensation should be used when managerial effort is not observable. The difficulty of observing managerial effort is especially a problem in financial intermediaries where human judgment and skill are key determinants of firm profitability. These firms face the classic principal-agent problem at many levels within the organization.[32]

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[31] In this illustration, the diversified firm simply makes more money per unit than the collection of stand-alone units, but units operated stand-alone are still viable (have a positive NPV.) However, in a competitive product market, the gains from diversification might be passed on to customers in the form lower fees and spreads. In that case, the individual business units would lose money if operated stand-alone, and their viability depends on being operated within a diversified firm.

[32] For a discussion of this literature, see Hart and Holmström (1987) and Milgrom and Roberts (1992).

In the standard principal-agent model, profits are a function of managerial effort, and management incurs disutility of effort. The larger the performance-sensitivity of compensation, the higher the level of effort, but the greater the risk that must be borne by management. Managerial risk aversion is a deadweight cost that must be covered by operating profits, and that generally will interact with the agency costs borne by outside stakeholders.

Consider a simple extension of the model developed thus far, beginning with a firm engaged in a single business. Let e denote managerial effort, and let $W(e) = \frac{1}{2}e^2$ represent management's disutility of effort. W(e) is an indirect expense to the firm. Let X denote firm operating profits before incentive payments to management and before making distributions to shareholders or receiving claims due on contract guarantees. In the model of the previous section, $X = \mu(1+r) + E(1)$, where E is the hedging error. X is normally distributed with present value μ and variance σ^2. Assume here that the firm has hedged all systematic risk so that X reflects only idiosyncratic project risk.[33] Suppose that effort affects the value of profits according to $\mu(e)$, where $\mu(.)$ in increasing and concave. Though not required for the key result to hold, assume for concreteness and to obtain a simple closed-form solution that $\mu(e) = be$, where b is a constant. Assume further that the variance of returns from balance-sheet positions can be monitored and controlled so that effort does not directly affect the variance of profits. Management's disutility for risk is $\frac{1}{2}\lambda V$, where V is the variance of management's compensation. Effort is not observable, although the functions W(e), $\mu(e)$, and managerial risk aversion, are known. The first-best level of effort is that which maximizes $\mu(e) - W(e)$, and is given by $e = b$. The first-best level of profits is thus $\mu = b^2$.

Finally, assume that management's compensation takes the form of a linear contract consisting of a base salary plus a share θ of operating profits. Linearity of the contract means that management shares proportionally in the downside as well as the upside. Real world institutional settings in which management bears contractual downside risk include general

[33] See Jin (2001) for a model of pay-performance sensitivity in which agents bear systematic as well as non-systematic risk.

partnership forms, incentive-fee clawback structures, and compensation arrangements involving low base pay.

Management values performance-related pay according to its certainty equivalent: $\theta\mu - W(e) - \frac{1}{2}\lambda\theta^2\sigma^2$. The level of effort that maximizes this certainty equivalent is given by $e = \theta b$, and the value of operating profits (before payouts to management) is thus $\mu = \theta b^2$. This is a second-best outcome unless $\theta = 1$, the case in which management is paid all of the profits and bears the whole risk of the firm.

Because management absorbs a fraction θ of the risk of the firm, the firm purchases proportionately less in the way of guarantees written on customer contracts, and there is proportionately less exposure to tax and agency costs between the firm and the outside capital markets. The firm also holds a proportionately smaller amount of initial cash. The firm's deadweight cost of risk capital thus is reduced to $(1-\theta)kR$, where R is the risk capital of the business before compensation payments. Internal plus external deadweight costs sum to $(1-\theta)kR + \frac{1}{2}\lambda\theta^2\sigma^2$. If management absorbs all of the profits ($\theta = 1$), managerial risk aversion represents the only deadweight cost.

The net present value of creating the firm is

NPV = Present value of operating profits
less disutility of effort less deadweight costs

$$= \mu(\theta b) - W(\theta b) - \{(1-\theta)kR + \tfrac{1}{2}\lambda\theta^2\sigma^2\}$$

Maximizing this expression over $\theta \leq 1$ yields the optimal sharing arrangement between the firm and management:

$$\theta^* = \min\{1, (b^2 + kR)/(b^2 + \lambda\sigma^2)\}$$

The numerator $(b^2 + kR)$ in this expression shows that two effects determine the performance sensitivity of compensation—an incentive effect (related to b, the sensitivity of profits to effort) and a risk sharing effect. The risk sharing effect is large when external agency costs (kR) are high relative to managerial risk aversion $(\lambda\sigma^2)$. In particular, firms should be entirely "owned" by management when $kR \geq \lambda\sigma^2$. Conversely, intermediaries with external agency costs that are low relative to managerial risk aversion should have significant outside ownership.

These results were obtained for the specific case of $\mu(e) = be$. When $\mu(.)$ is a general concave function of effort, it is straightforward to show that θ^* is increasing in kR and therefore that the conclusions hold more generally.

Performance sensitivity of division-level compensation

A further consequence of this theory is that the performance sensitivity of compensation for division-level management should be different in diversified firms than in units operated stand-alone.

Consider a firm composed of N symmetric divisions whose operating profits X_i are uncorrelated, each with standard deviation σ. Let e_i and $W_i(e_i) = ½e_i^2$ respectively denote the effort and disutility of effort of the management of division i (herein after called "management i".) As before, suppose that the value of divisional operating profits is related to effort according to $\mu_i(e_i) = be_i$. Let the disutility of risk of management i be $½\lambda V_i$.

Suppose now that management i's incentive compensation takes the form of a share θ_{iF} of firm-wide operating profits plus a share θ_{iD} of the operating profits of division i. The performance-based compensation of management i is thus $\theta_{iD}X_i + \theta_{iF}\Sigma X_j$, and the economic sensitivity of this package to the performance of division i is = $\theta_{iD} + \theta_{iF}$. The economic sensitivity θ_i measures the contractual sensitivity θ_{iD} plus the sensitivity to X_i implicit in the exposure of manager i's compensation to firm-wide operating profits. Holding fixed θ_i, θ_{iF} is the economic sensitivity of manager i's compensation to the performance of the units *other* *than* i. Firm-wide operating profits net of incentive-compensation payments sum to $\Sigma X_i(1 - \theta_{iD} - \Sigma\theta_{jF})$.[34]

Management i's certainty equivalent of compensation is given by:

$$(\theta_{iD}+\theta_{iF})\mu_i(e_i) + \Sigma_{j\neq i}\theta_{iF}\mu_i(e_i) - W_i(e_i) - \tfrac{1}{2}\lambda \mathrm{Var}\{\theta_{iD}X_i + \theta_{iF}\Sigma X_j\}$$

and the net present value of creating the firm is

$$\Sigma_i[\mu_i(e_i) - W_i(e_i) - \tfrac{1}{2}\lambda \mathrm{Var}\{\theta_{iD}X_i + \theta_{iF}\Sigma X_j\}] - \text{Std Dev}\{X_i\,(1 - \theta_{iD} - \Sigma\theta_{jF})\}\; kR_U\,/\sigma$$

where R_U is the stand-alone risk capital of an individual division.

By symmetry of the N operating divisions, the optimal compensation arrangements will be the same across divisions. Thus, for all i, $\theta_{iF} = \theta_F$, $\theta_{iD} = \theta_D$, and $\theta = \theta_F + \theta_D$. Since the profits of the divisions are assumed uncorrelated and have the same standard deviation, the variance of divisional incentive compensation is $(\theta^2 + (N-1)\theta_F^2)\sigma^2$; and the variance of firm-wide operating profits net of incentive-compensation payments is $(1 - \theta - (N-1)\theta_F)^2\sigma^2$.

Solving for the optimal compensation schedule in the same manner as the single division case yields:

$$\theta^* = \theta^*_D + \theta^*_F = (b^2 + kR_U/\sqrt{N})/(b^2 + \lambda\sigma^2)$$

$$\theta^*_F = (kR_U/\sqrt{N})/\lambda\sigma^2$$

and

[34] This particular structure of the incentive compensation package is chosen for expositional convenience. In most real-world settings, the portion of compensation tied to firm-wide profits is expressed in terms of profits calculated net of divisional compensation payments. In the case of symmetric business units considered here, the optimal solution to this latter formulation follows from a simple transformation: If θ'_{iF} denotes management i's share of firm-wide profits net of divisional compensation payments, then θ'_{iF} and θ_{iF} are related through $\theta_{iF} = (1 - \theta'_{iD})\theta'_{iF}$. The optimal economic sensitivity

$$\theta^*_D = (1 - \theta^*_F)b^2/(b^2 + \lambda\sigma^2)$$

in the range where $\theta^* \leq 1$.

In this solution, manager i's economic sensitivity of pay (θ^*) to the performance of division i has the same functional form as in the single division case except that the risk sharing effect now is between the diversified firm's *per division* cost of risk capital ($kR_U/\sqrt{N}$) and managerial risk aversion ($\lambda\sigma^2$). As the firm becomes more diversified, the per division deadweight cost of risk capital decreases (with $1/\sqrt{N}$) while the riskiness of profits at the division level remains constant. Managerial risk aversion thus takes on greater relative importance within the diversified firm, and manager i's economic sensitivity of pay to the performance of division i is decreasing in N.

Note that since θ^*_F is the economic sensitivity of manager i's compensation to the performance of the units other than i, and since manager i in this model can control the performance only of unit i, tying divisional manager compensation to the profits of other divisions serves a purely risk-sharing function. Thus, θ^*_F is not a function of the sensitivity of profits to effort, and θ^*_F decreases to zero as N gets large.

Note also that, since θ^*_F is decreasing in N, the contractual sensitivity of compensation to divisional performance (θ^*_D) is increasing in N. This is an artifact of the manner in which the contract has been specified. As already mentioned, the economic sensitivity of compensation to divisional performance ($\theta^* = \theta^*_D + \theta^*_F$) is decreasing in N. That the contractual sensitivity of compensation to divisional performance might increase as the firm becomes more diversified suggests that, for the purpose of empirical testing, contractual sensitivity might be a poor proxy for economic sensitivity.

θ'_i to the profits of division i equals θ_i, and θ'_{iD} and θ'_{iF} behave directionally as functions of N in the same way as θ_{iD} and θ_{iF}.

Finally, if the performance sensitivity of divisional compensation in diversified firms is lower than in units operated stand-alone, then managerial effort also is lower and the diversified firm earns lower profits per division before compensation payments and deadweight costs. It seems paradoxical then that the diversified firm might be worth more than the sum of its units operated stand-alone. The answer is that diversification reduces the deadweight cost of risk capital by enough to offset the commensurate reduction in managerial effort. Inclusive of all deadweight costs, the NPV per division within the diversified firm evaluates to:

$$\text{NPV per division} = \tfrac{1}{2}\theta^{*2}(b^2 + \lambda\sigma^2) - \lambda\sigma^2\theta^*_F + \tfrac{1}{2}(N-1)\lambda\sigma^2\theta^{*2}_F$$

which is increasing in N for $\theta^* < 1$.[35] Over and beyond this effect, and as illustrated previously, the diversified firm is also worth more because it can operate its divisions on a larger scale.

Conclusions

This paper has developed a model in which opaque financial intermediaries bear deadweight agency and tax costs related to firm-wide risk, and it has explored normative implications of the model for capital budgeting, risk management, and incentive compensation.

Empirical predictions of the model are that opaque financial firms will diversify across businesses that bear similar deadweight costs. These divisions experience a lower cost of risk capital which has the effect of creating more profitable investment opportunities at the margin and therefore enabling the divisions to operate on a larger scale. The model also shows that empirical studies of conglomerate discounts that do not adjust for the effect of expanded investment opportunity sets might spuriously conclude that diversification is costly.

[35] The derivative of this expression with respect to N is $\tfrac{1}{2}\lambda\sigma^2\theta^*_F(\theta^*_F+1-\theta^*)/N$.

The model of this paper further shows that incentive-based compensation serves a risk-sharing function that reduces opaque financial intermediaries' external deadweight costs of risk capital. This benefit is less pronounced for diversified firms, and the theory thus predicts that the economic sensitivity of divisional manager compensation to both divisional performance and firm-wide performance should be lower in diversified firms than in comparable units operated stand-alone.

With respect to future research, the model needs to be extended in at least several ways. First, there is a need for coordinated decision-making driven by external deadweight costs being a function of firm-wide risk. In its present form, the model is silent on how coordination of decisions should take place within the firm. Second, this is a one-period model, and it needs to be extended to a multi-period context in which the attractiveness of taking both short-term and long-term positions (such as illiquid bank loans) can be evaluated. A third subject for future research is the measurement and estimation of deadweight costs. In an efficient market, these costs cannot be inferred from returns data (consistent with the model of this paper). Instead, estimation must be based on firm valuation, perhaps through the use of P/E, P/B and other multiples.

Finally, the model does not account for frictions created specifically by corporate diversification—such as reduced monitoring effectiveness by headquarters and other diseconomies of scope. A fundamental question therefore is to what extent the effects modeled here are counterbalanced by inefficiencies associated with corporate diversification.

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Figure 1 Optimal capital structure: A trade-off between the deadweight cost of equity financing and the deadweight cost of insurance

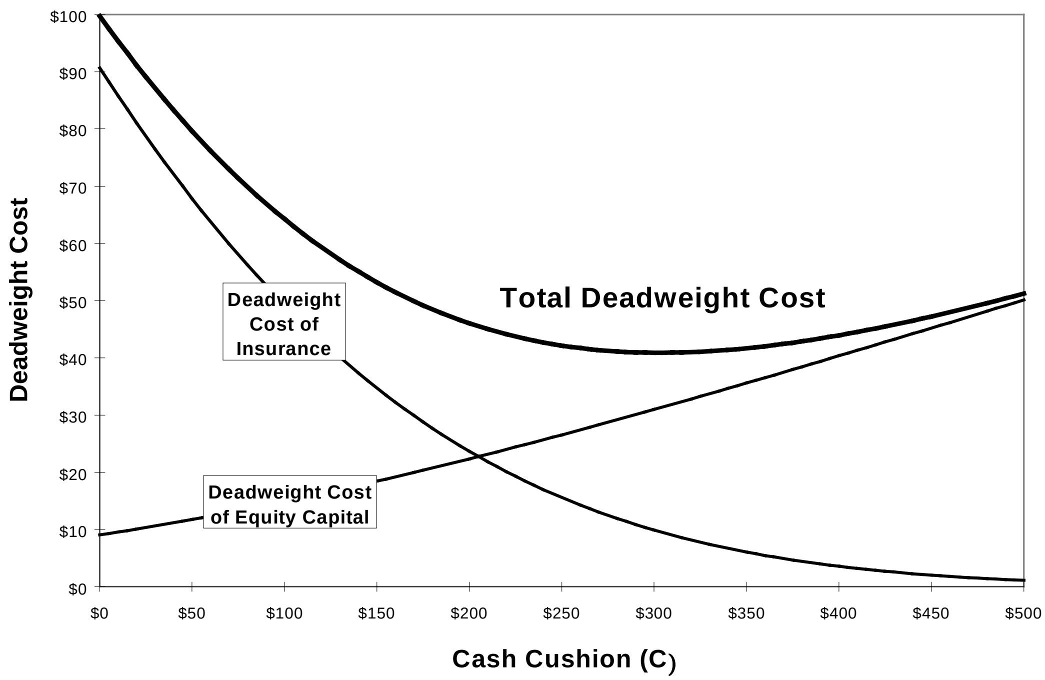

Figure 2 Deadweight cost of risk capital (k) and optimal cash cushion (varying d, m=100%)